2013 Focus Issue:
Developing & Empowering
Teacher Leaders

Making Shifts toward

This guide will support teacher leaders who work to cultivate classrooms where developing mathematical practices is a daily, intentional goal.

Proficiency

By Maggie B. McGatha
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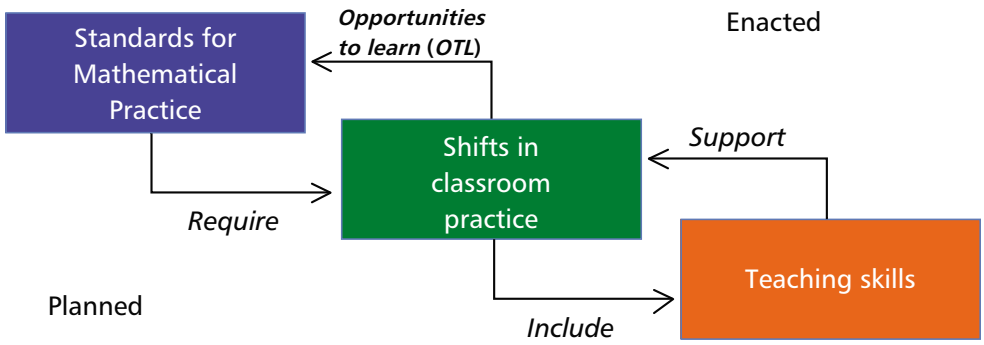
We all want students to become mathematically proficient. The road leading there may be long, but the Common Core State Standards (CCSS) for Mathematical Practice (CCSSI 2010) provide a clear description of what the destination looks like (http://www.corestandards.org/Math/Practice). As a mathematics teacher leader, you may see lessons and consider to what extent they present opportunities for students to demonstrate the Standards for Mathematical Practice. Think, for example, about a lesson you have seen on subtraction with regrouping or a lesson on the properties of addition. How might these operations be taught in a way to nurture the Mathematical Practices in students?

Here we share a vignette from a third-grade classroom. As you read it, look for examples of students demonstrating the Standards for Mathematical Practice. Students begin by reading the Add Three to Both conjecture task:

FIGURE 1

A teacher leader determines the most effective way to support teachers in shifting classroom practices to provide the best environment for helping all students become mathematically proficient.



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In a subtraction problem, you can add three to both numbers (the minuend and the subtrahend), and you will get the same answer.

Prepare an explanation that will convince your classmates whether this conjecture is true or false.

Anissa: That doesn't make any sense.
Ms. Travers: Let's look at an example. How about 82 – 37. Now add 3 to 82, and add 3 to 37. What do you have?
Jeanette: 85 – 40
Travers: Solve both problems. Is the answer the same?
Students: Yes.
Travers: Let's look at the statement again [*reading the statement again, she points to the example*].
Students: It works.
Travers: So, it is true for this example. But, you have to be able to convince us that it is *always* true—true for any numbers. Work with your partner to design a way to show this will (or will not) be true for any numbers.
[*Students do their calculations.*]
Travers: Let's share your thinking. Adam?
Adam: We tried it for three different problems, and it always works.
Travers: What do you think? Do these examples convince you it will always be true?
Students: Yes!
Travers: These examples are good for *testing* if the statement is true. However, to *prove* it, we want to be able to say it works for all numbers. Work with your partner to try to *prove* it is true.
[*Students work through the task again.*]
Travers: Let's hear your ideas. Annie?
Annie: It's true no matter what. If you add something to the top and add the same thing to the bottom, you are really just taking it away again. So, you added and subtracted the same amount, and so you get the same answer.
Ethan: Oh, right, I get it—it is added and subtracted again.
Jay: I did it on a number line [*holding up his paper*]. If you start here, say 67, and go up 3 to 70, then you just go back down 3, back to 67, and subtract the second number. You have the same problem you started with, no matter where you start.
Travers: We have two new explanations. Do these explanations convince you that this conjecture is true for *all* numbers?

This excerpt gives us a glimpse of several Standards for Mathematical Practice in action. Students were designing an approach to show whether the statement was true, which required perseverance (MP1), quantitative reasoning (MP2), and constructing a viable argument (MP3). To help them reason, students selected tools (e.g., a number line) (MP5) and looked for regularity in repeated reasoning (MP8).

To what might we credit this student's reasoning and sense making? Certainly, the teacher plays a critical role. The teacher selects the task, sets up the task, considers what mathematics relates to this topic, determines questions to ask, and monitors students. So, how do teachers refine and develop *their* skills? The mathematics teacher leader plays a critical role. He or she selects foci for professional development, plans coaching questions, and uses data to monitor progress. Here we share a framework that can guide and support mathematics teacher leaders as they work with teachers to create classroom environments where cultivating the Mathematical Practices is a daily, intentional goal.

The framework

The Leading for Mathematical Proficiency Framework (Bay-Williams et al. 2013) has three components (see **fig. 1**):

1. The Standards for Mathematical Practice
2. Shifts in classroom practice
3. Teaching skills

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The LMP framework guides development of a culture where students learn to engage in and develop facility with the Mathematical Practices.

We will briefly describe each component of the LMP framework as an overview and then focus more in depth on the second component, the shifts in classroom practice.

Standards for Mathematical Practice

The LMP framework begins with student outcomes, that is, what we want students to know and be able to do. We want students to be able to demonstrate the Mathematical Practices. This is a good place to begin having conversations with teachers, such as, "What does Standard 3 ['Construct viable arguments and critique the reasoning of others,' CCSSI 2010, p. 6] look like in action?" What might it look like in a lesson on multiplication? In an assessment?

Opportunities to demonstrate

Having conversations with teachers about the Standards for Mathematical Practice is a good place to start, but it is not enough. For students to demonstrate the practices, teachers must provide regular opportunities to develop these proficiencies. A focus question to ask in this second component of the LMP framework is, "What shifts in classroom practice might be necessary for students to have the opportunity to demonstrate a particular Standard?" This is a reflective question for math teacher leaders, as well as an explicit question to pose as teachers determine their own professional learning priorities.

Teaching skills

Teaching skills focus on one aspect of teaching (e.g., questioning) that can support progress on one or more shifts. The focus question now becomes, "What knowledge or skills will enable and support this teacher in shifting classroom practices?" The answer could be any or several of the following: questioning, strengthening content knowledge, differentiating instruction, and so on. Focusing on specific teaching skills presents an opportunity for targeted discussion, learning, and documenting of practices that can then be connected back to the shifts and to developing the Mathematical Practices.

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Shifts in classroom practice can better connect teaching to student learning.

The LMP framework proposes a process for planning professional learning opportunities: Start at the top with exploring the Mathematical Practices; then, identify shifts in classroom practice that offer opportunities for these practices to develop; and finally, focus on teaching skills to support the shifts in classroom practice. At the center of the LMP framework are the Shifts in Classroom Practice (see **fig. 2**). These seven shifts represent a synthesis of research and standards' recommendations that connect teaching to student learning (e.g., Hiebert and Grouws 2007; NCTM 1991, 2007). We now focus on this aspect of the framework, highlighting (1) what each shift looks like and (2) questions to engage teachers in dialogue related to that shift.

Shifts in classroom practice

Classroom practices described on the right side of the Shifts in Classroom Practice (Bay-Williams et al. 2013) continua (see **fig. 2**) illustrate that the shifts are necessary in providing optimal learning opportunities for students to demonstrate the Mathematical Practices and become mathematically proficient. The shifts are not intended to be an evaluation tool but rather a self-assessment tool. Teachers can identify one shift they feel will make the most difference in their students' learning, or they can work on several of the shifts, assessing where they are and monitoring their own progress. A professional learning community (PLC) might identify one shift and keep it in the forefront as its participants select tasks, teach learners, and analyze student work. In the following sections, we briefly share important features of each shift, along with coaching questions to focus teacher attention on essential characteristics of the shift.

Shift 1: Toward differentiated instruction

The first shift is twofold: (1) instruction should be differentiated to meet the needs of all learners (this could mean varying the amount of structure in a task), and (2) all students have the same learning outcomes. Using the opening vignette as an example, all students must know strategies for subtraction as well as must be able to determine when a process will always work. This shift targets students' individual readiness and strengths and ensures that all students reach the intended learning goal.

Coaching questions

- What are some ways that you currently differentiate instruction?
- Identify some specific needs of students in your class. In what ways might this lesson be adapted to meet those specific needs?
- What strategies might you use to include higher-level thinking and complex tasks and ensure that all students have access and opportunity to engage in solving them?

Shift 2: Toward a community of learners

A key to the second shift is asking who has authority in the classroom. Who confirms or contests answers to problems? The teacher may ask students if they agree or have different answers or may encourage student-to-student discussion of strategies in other ways. The idea of establishing a community of learners can

FIGURE 2

Although the shifts are a self-assessment tool, a PLC might consider them when selecting tasks, instructing learners, and analyzing student work.

Shifts in Classroom Practice

Shift 1: From *same instruction* toward *differentiated instruction.*

Same instruction for all students.	Differentiated instruction but same learning outcomes for all students.

Shift 2: From *students working individually* toward *community of learners.*

Students work individually on tasks and seek feedback from teacher on reasonableness of strategies and solutions.	Community of learners where students hear, share, and judge reasonableness of strategies and solutions.

Shift 3: From *mathematical authority coming from the teacher or textbook* toward *mathematical authority coming from sound student reasoning.*

Correctness of solutions is determined by seeking input from teacher or textbook.	Correctness of solution is based on reasoning about the accuracy of the solution strategy.

Shift 4: From *teacher demonstrating "how to"* toward *teacher communicating expectations for learning.*

Teacher demonstrates the way in which to solve a problem and helps students in solving the problem in that way.	Teacher facilitates high-level performance by sharing learning goals and expectations for products that demonstrate learning.

Shift 5: From *content taught in isolation* toward *content connected to prior knowledge.*

Content presented independent of its connections to what has been previously learned.	Content presented in ways in which explicit attention is given to making connections among mathematical ideas.

Shift 6: From *focus on correct answer* toward *focus on explanation and understanding.*

Discussions and classroom routines focus on student explanation of how they solved a task and whether it is correct.	Discussions and classroom routines focus on student explanations that address why an answer is (or is not) correct.

Shift 7: From *mathematics made easy for students* toward *engaging students in productive struggle.*

Mathematics is presented in small chunks and help is provided so that students reach solutions quickly and without higher-level thinking.	Teacher poses tasks and challenges students to persevere and attempt multiple approaches to solving problems.

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Math coaches can help teachers make the shifts in classroom practice by asking questions that focus attention on essential characteristics of the instructional changes that are required.

be confused with mistakenly thinking that the teacher does not explain how to solve a problem. In fact, a teacher's input and contributions are essential to the learning. However, teacher moves are more subtle and complex than confirming a correct answer, as we see when Ms. Travers, the teacher in the vignette, pressed students by clarifying the difference between testing and proving.

Coaching questions

- List several important aspects of a community of learners.
- What might be some ways to encourage students to share their thinking? How do you decide when to use those strategies?
- How might you go about developing a community of learners where students are comfortable questioning (or being questioned by) one another?

Shift 3: Toward mathematical authority coming from sound student reasoning

As a class moves toward a community of learners (shift 2), the mathematical authority moves to the community (shift 3). The teacher and textbook can serve as resources, but the correctness of an answer should be grounded in reasoning. How might students respond to this question: How do you know if this answer is correct? In a classroom that reflects the right side of this continuum, a student is likely to offer a mathematical rationale, as did the students in Travers's class when they devoted time to developing their own explanation for why the Add Three to Both Conjecture was true. This shift is sometimes unfamiliar and uncomfortable for teachers, but with support, they can see how empowering it can be for students.

Coaching questions

- What could be some benefits of students determining for themselves (individually or collaboratively) the correctness of their thinking?
- Which strategies have you used to encourage students to reason and think for themselves?
- What might get in the way of students reasoning and thinking for themselves?

Shift 4: Toward teachers communicating expectations for learning

This shift is about opening possibilities for students. Instead of the teacher showing students how to solve tasks and expecting them to solve them only in that way, this shift suggests that teachers should set high standards, should accept multiple ways of getting there (as Travers did by clarifying that she wanted students to prove the conjecture was true), and should encourage student-generated approaches. Communicating expectations for students can include such things as sharing learning goals, providing rubrics, or creating lists of performance indicators.

Coaching questions

- What processes do you use in determining students expectations for a [task, project, assignment, or unit of study]?
- What are some strategies you use to communicate expectations for learning?
- What are some alternative strategies to teaching [a topic] beyond teacher explanation?
- How might you offer structure or guidance for a task without giving away exactly how to do it?

Shift 5: Toward content connected to prior knowledge

Shift 5 is all about making connections. Teaching topics in isolation usually results in students having shallow understandings of mathematics. *Curriculum Focal Points* (NCTM 2006) and the overview page of each grade level in CCSSM effectively describe the interconnectedness of mathematics. Learning progressions (e.g, http://ime.math.arizona.edu/progressions/) highlight the developmental nature of mathematics. For mathematics instruction to be truly meaningful and lasting, we have to support students in building on their own understanding and making connections among mathematical ideas.

Coaching questions

- What might be some benefits for students in making connections between mathematical ideas? What are some for you, the teacher?
- List ways that you make connections between mathematical content. So, what might these look like for students?
- What might a lesson look like that accesses students' prior knowledge?

Shift 6: Toward a focus on explanation and understanding

"Ours is not to reason why, just invert and multiply." This mantra would fit on the left end of the sixth continuum. We must move *away* from students simply explaining steps they completed to solve a (known) procedure and *toward* rich discussions that focus on how, why, and when a certain approach might work. This is the kind of real thinking that will be required of students when they are living outside the mathematics classroom. Travers is pressing for such explanations as she gives students time to work with a partner in deciding why the conjecture is true in the opening vignette. Lessons must furnish opportunities for students to explain their thinking and reasoning.

Coaching questions

- What might be some benefits to students explaining why or how an algorithm works?
- What supports do you need as a problem solver to explain your thinking? To learn how to explain their thinking to others, what kind of supports might students need?
- How can you communicate to students that their reasoning is valued and important?

Shift 7: Toward engaging students in productive struggle

A strong research finding in terms of developing conceptual knowledge is to engage students in productive struggle (Hiebert and Grouws 2007), like Travers asking her students to prove the conjecture. Many teachers chose this profession because of a desire to help others, but helping has too often been equated with explaining and showing. In fact, explanations can prevent students from learning to think on their own. Allowing students to struggle (productively) is helping them—to build connections among ideas, to feel a sense of accomplishment, to apply their own knowledge, and to develop a strong self-efficacy for being able to do mathematics—what *Adding it Up* (NRC 2001) describes as a "productive disposition" (p. 116).

Coaching questions

- What might be some benefits of allowing students to engage in productive struggle?
- What might it look like in a specific lesson if you were allowing students to engage in productive struggle? What adjustments might need to be made by you and by the students?
- What are some ways that you challenge students to persevere on mathematical tasks?
- In your classroom, how do you deal with multiple approaches to solving tasks?

The most effective support

The "job description" of the mathematics teacher leader is to determine the most effective way to support teachers in shifting classroom practices in ways that will provide the best environment for helping all students become mathematically proficient (see **fig. 1**). The Leading for Mathematical Proficiency framework can do the following:

- Focus explicit attention on outcomes for students
- Connect CCSS to research on effective classroom practice
- Guide decision making related to professional learning opportunities

Collectively, the three components of the LMP framework support the development of a cohesive and focused professional learning plan that equips teachers with the skills they need to engage students in ways that foster the development of the Mathematical Practices.

At the center of the LMP framework, the shifts—grounded in research and standards—provide opportunities for reflective practice. Reflective questions focused on the shifts suggest a beginning to the kind of dialogue that can occur in—

- self-reflection;
- coaching conversations;
- PLC discussions; and
- lesson study.

Four hundred years ago, Galileo said, "We cannot teach people anything, we can only help them discover it within themselves." Today, this quote still captures the essence of teacher learning. The mathematics teacher leader's goal is to engage teachers in productive professional conversations that help them discover more effective ways to support their students' learning. The framework and shifts presented here can support this goal.

Common Core Connections

SMP 1 **SMP 5**
SMP 2 **SMP 8**
SMP 3

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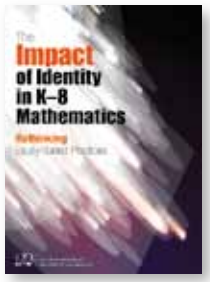

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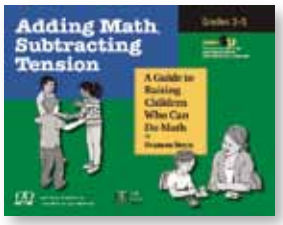

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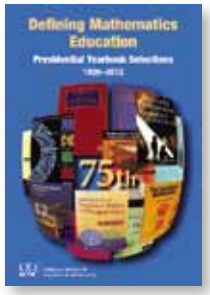

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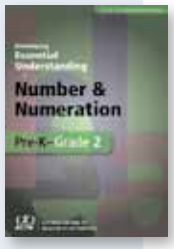

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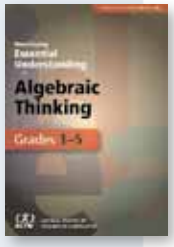

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